Review for Final Exam

MAT 233

May 12–13, 2011

Logistics

- Tuesday, 8:00–10:00 AM, Old Main 320
- Resources: MATLAB/MuPad, calculator
- Space will be provided on the test form; no need for extra paper
- Exams should be graded by Wednesday afternoon, semester grades posted by Thursday
- Exams kept on file with Kristine Wood until October 2011

Structure and Philosophy

- 150 points, $\approx 17\%$ of semester grade
- Comprehensive (Chapters 1, 2, 3, 5, 6)
- Components:
  - Multiple choice (like clicker questions) — roughly 15%
  - True/False + explain — roughly 20%
  - Proofs — roughly 15%
  - Problems — roughly 50%

Preparation

- Use objective measures of competency — DO THINGS, don't rely on "feelings of understanding"
- Use the competency lists
- Problems from Lay textbook; in-class problems from workshops done individually; make up your own problems to work; look in other books
- Start NOW
- Realistic goal = 1 chapter per night from Thursday through Monday

Which of the following is an elementary row operation?

(A) Replace a row with itself plus a multiple of another

(B) Replace a row with a multiple of itself plus another

(C) Swap two rows

(D) Rescale a row by a nonzero scaling factor

(E) All of the above

(F) (A), (C), and (D)

(G) I don't know

Any system of n linear equations in n variables has at most n solutions.

(A) True

(B) False

(C) I don't know

For the next several questions, let

$$A = \begin{bmatrix} 2 & 0 & 6 \\ -1 & 8 & 5 \\ 1 & -2 & 1 \end{bmatrix}$$

The reduced echelon form of A is

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Suppose $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are the columns of A. The number of vectors in $\mathrm{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$

(A) Equals 2

(B) Equals 3

(C) Is infinite

(D) Cannot be determined

(E) I don't know

$$A = \begin{bmatrix} 2 & 0 & 6 \\ -1 & 8 & 5 \\ 1 & -2 & 1 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

The vector $\mathbf{b} = [2\ 2\ 2]^T$ is in $\mathrm{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

(A) True

(B) False

(C) I don't know

$$A = \begin{bmatrix} 2 & 0 & 6 \\ -1 & 8 & 5 \\ 1 & -2 & 1 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

The pivot columns of A

(A) Form a basis for $\mathbb{R}^3$

(B) Form a basis for $\mathbb{R}^2$

(C) Are linearly independent

(D) Both (A) and (C)

(E) Both (B) and (C)

(F) I don't know

For any matrix A (not just the one from the last few questions), the equation $A\mathbf{x} = \mathbf{0}$ has a solution if and only if there are no free variables.

(A) True

(B) False

(C) I don't know

If a set of vectors is linearly dependent, then each vector in the set is a scalar multiple of the others.

(A) True

(B) False

(C) I don't know

Suppose the equation $A\mathbf{x} = \mathbf{0}$ has only the trivial solution. Then

(A) A is invertible

(B) The columns of A are linearly independent

(C) None of the columns of A can equal the zero vector

(D) All of the above

(E) (B) and (C)

(F) I don't know

Suppose $AB = C$ and C has 2 columns. Then A has 2 columns.

(A) True

(B) False

(C) I don't know

If A and B are $n \times n$, then $(A+B)(A-B) = A^2 - B^2$.

(A) True

(B) False

(C) I don't know

Suppose B is a matrix whose third column is the zero vector. Let A be another matrix such that AB is defined. Then

(A) The $(3,3)$-entry of AB equals 0

(B) The third row of AB is the zero vector

(C) The third column of AB is the zero vector

(D) (A) and (B)

(E) (A) and (C)

(F) I don't know

Let E be an $n \times n$ elementary matrix. The number of nonzero entries of E

(A) Could equal n

(B) Could equal $n + 1$

(C) Could equal $n - 1$

(D) All of the above

(E) (A) and (B)

(F) (A) and (C)

(G) Cannot be determined

(H) Can be determined, and it's none of the above

(I) I don't know

Suppose A is an 5×5 invertible matrix. Then

(A) The equation $A\mathbf{x} = \mathbf{b}$ has a solution for each $\mathbf{b} \in \mathbb{R}^5$.

(B) The columns of A are linearly independent.

(C) The rows of A are linearly independent.

(D) All of the above

(E) (A) and (B)

(F) (A) and (C)

(G) I don't know

The matrix A has an LU factorization

(A) If and only if A is square

(B) If and only if A is invertible

(C) If and only if the columns of A form an orthogonal set

(D) If and only if the eigenvalues of A are all distinct

(E) None of the above

(F) I don't know

Any linear combination of two solutions to the equation $A\mathbf{x} = \mathbf{0}$ is another solution to this equation.

(A) True

(B) False

(C) I don't know

$$A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix} \xrightarrow{RREF} \begin{bmatrix} 1 & -2 & 0 & -1 & 3 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The null space of A

(A) Contains exactly two vectors

(B) Contains exactly three vectors

(C) Has dimension 2

(D) Has dimension 3

(E) I don't know

The columns of a 2×3 matrix form a basis for the column space of the matrix.

(A) True

(B) False

(C) I don't know

Suppose U is the echelon form of A. Then the determinant of A equals the product of the diagonal entries of U.

(A) True

(B) False

(C) I don't know

Suppose A is $n \times n$ and $\det A = 2$. Then $\det(A^3)$

(A) Equals 2

(B) Equals 3

(C) Equals 6

(D) Equals 8

(E) Cannot be determined without more information

(F) I don't know

Suppose $\det A = 3$ and B is obtained from A by swapping two rows and rescaling the first row by a factor of 2. Then $\det B$ equals

(A) -6

(B) -3

(C) -2

(D) 2

(E) 3

(F) 6

(G) None of the above

(H) I don't know

Every square matrix has at least one real eigenvalue.

(A) True

(B) False

(C) I don't know

Suppose that $L : \mathbb{R}^2 \to \mathbb{R}^2$ is the linear transformation that flips vectors in $\mathbb{R}^2$ over the line $y = 2x$. Suppose A is the standard matrix for L. Then

(A) $\lambda = 1$ is an eigenvalue of A

(B) $\lambda = -1$ is an eigenvalue of A

(C) $\lambda = 2$ is an eigenvalue of A

(D) Both (A) and (B)

(E) Both (A) and (C)

(F) Both (B) and (C)

(G) I don't know

Suppose $\lambda = -1$ is an eigenvalue for the matrix B. Then an eigenvector corresponding to this eigenvalue

(A) Is a nontrivial solution to $B\mathbf{x} = \mathbf{0}$

(B) Belongs to the null space of B

(C) Belongs to the null space of $B + I$ where I is the identity matrix

(D) Belongs to the null space of $B - I$ where I is the identity matrix

(E) I don't know

Suppose the matrix M has exactly two eigenvalues, $\lambda = 2$ and $\lambda = -3$. Then the characteristic polynomial for M is

(A) $\lambda^2 + \lambda - 6$

(B) $\lambda^2 - \lambda - 6$

(C) $\lambda^2 + 5\lambda + 6$

(D) $\lambda^2 - 5\lambda + 6$

(E) Cannot be determined without more information

(F) I don't know

An 3×3 matrix is diagonalizable if

(A) It has three linearly independent columns

(B) It has three linearly independent eigenvectors

(C) Its columns are an orthogonal basis for $\mathbb{R}^3$

(D) I don't know

The least-squares solution $\hat{\mathbf{x}}$ for a linear system $A\mathbf{x} = \mathbf{b}$ can be calculated by

(A) $\hat{\mathbf{x}} = (A^T A)^{-1} A^T \mathbf{b}$

(B) $\hat{\mathbf{x}} = R^{-1} Q^T \mathbf{b}$ where $A = QR$ is a QR factorization

(C) All of the above

(D) I don't know