

Workshop on Determinants  
MAT 233, Linear Algebra  
April 8, 2011

Work on the following problems in groups of 2. Please submit a rough draft of your work by the end of the class period today, and submit a final draft of your work by class time on Monday. Both the rough and final drafts here may be either L<sup>A</sup>T<sub>E</sub>X'ed or neatly handwritten.

TASK 1: Calculate the determinant of the following matrix:

$$\begin{bmatrix} -19 & -19 & -5 & 7 & -10 & -10 & 14 & -18 & 0 & 12 \\ 14 & -9 & 11 & 6 & 0 & 13 & 3 & 1 & 0 & -8 \\ 18 & -19 & 12 & -14 & 8 & -11 & 2 & 11 & 0 & 1 \\ 7 & -17 & -13 & -16 & 16 & 18 & 17 & 18 & 0 & -14 \\ 11 & 13 & 0 & 0 & 19 & -6 & -9 & -15 & 0 & 4 \\ 10 & 8 & -2 & 19 & 2 & -12 & 11 & 3 & 0 & -10 \\ -4 & -7 & 6 & -7 & -15 & -10 & 10 & -1 & 0 & 6 \\ 6 & 18 & 9 & 3 & -14 & 5 & -5 & -20 & 0 & 8 \\ -13 & -19 & 10 & -11 & -10 & -1 & 3 & -7 & 0 & 10 \\ 8 & -3 & -9 & 10 & 14 & -6 & -17 & -14 & 0 & -2 \end{bmatrix}$$

TASK 2: You've probably figured out by now that the command `det` in MATLAB will calculate determinants. Using this command and through experimentation with many examples, make a conjecture about the determinant of a square lower-triangular matrix. Your conjecture should be of the form:

The determinant of a square lower-triangular matrix is \_\_\_\_\_.

where the blank is filled in by an informative, logical, and mathematically correct statement. For example, "a number" would be a correct thing to say but not terribly informative. "equal to 0" would be informative but not correct. Make the conjecture such that it gives some real insight into how determinants of square lower-triangular matrices are calculated.

Once you have made your conjecture, prove that it is true for all  $2 \times 2$ ,  $3 \times 3$ , and  $4 \times 4$  lower-triangular matrices.