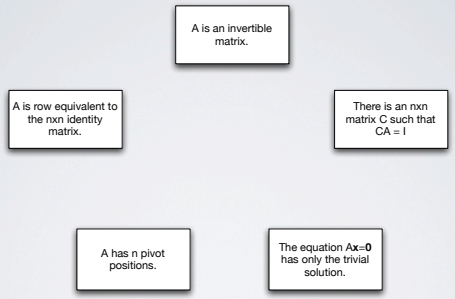
CHARACTERIZATIONS OF INVERTIBLE MATRICES

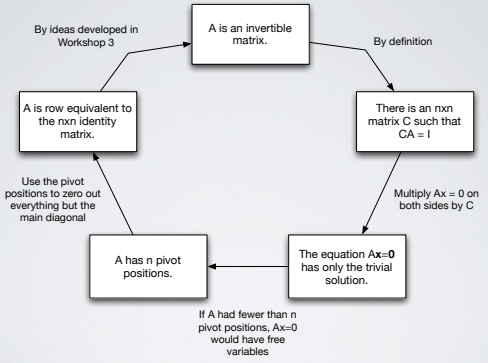
MAT 233 : Linear Algebra

1

A: nxn square matrix



2



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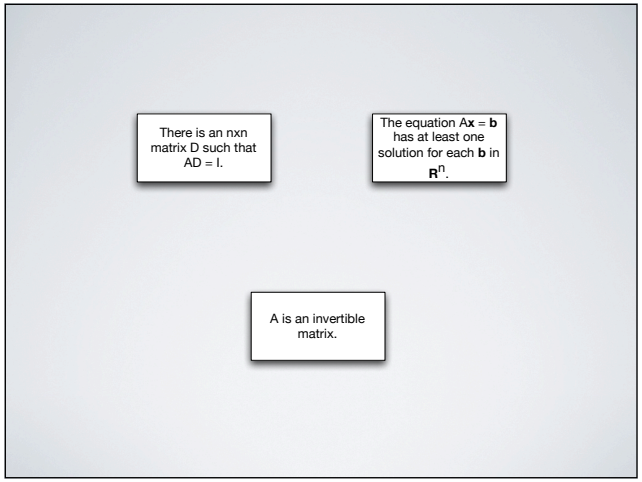
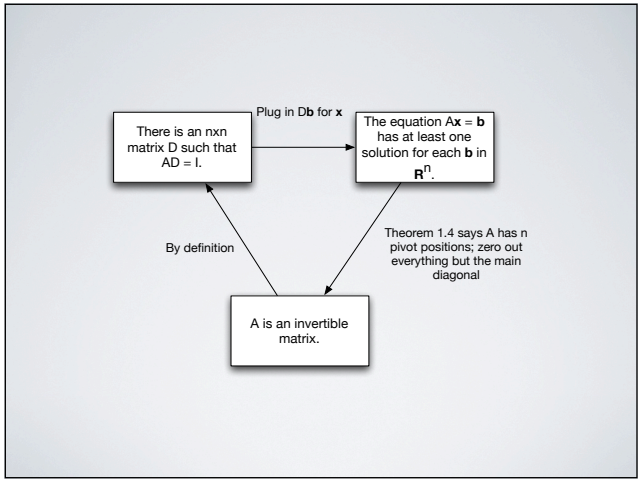

There is an nxn matrix D such that AD = I.
The equation A**x** = **b** has at least one solution for each **b** in $\mathbf{R}^n$.
A is an invertible matrix.

4


There is an nxn matrix D such that AD = I.
Plug in D**b** for **x**
The equation A**x** = **b** has at least one solution for each **b** in $\mathbf{R}^n$.
By definition
Theorem 1.4 says A has n pivot positions; zero out everything but the main diagonal
A is an invertible matrix.

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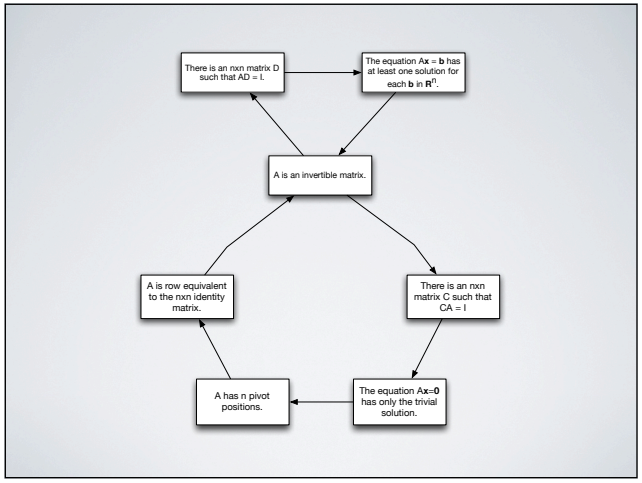
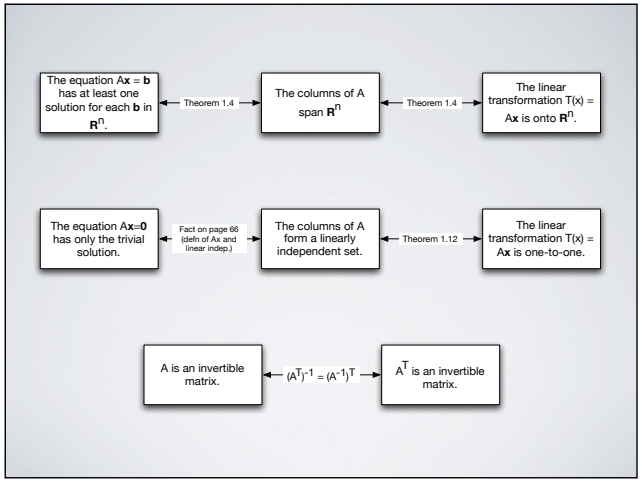

There is an nxn matrix D such that AD = I.
The equation A**x** = **b** has at least one solution for each **b** in $\mathbf{R}^n$.
A is an invertible matrix.
A is row equivalent to the nxn identity matrix.
There is an nxn matrix C such that CA = I
A has n pivot positions.
The equation A**x**=**0** has only the trivial solution.

6


The equation Ax = b has at least one solution for each b in $\mathbf{R}^n$.
Theorem 1.4
The columns of A span $\mathbf{R}^n$
Theorem 1.4
The linear transformation T(x) = Ax is onto $\mathbf{R}^n$.
The equation Ax=0 has only the trivial solution.
Fact on page 66 (defn of Ax and linear indep.)
The columns of A form a linearly independent set.
Theorem 1.12
The linear transformation T(x) = Ax is one-to-one.
A is an invertible matrix.
$(A^T)^{-1} = (A^{-1})^T$
A^T is an invertible matrix.

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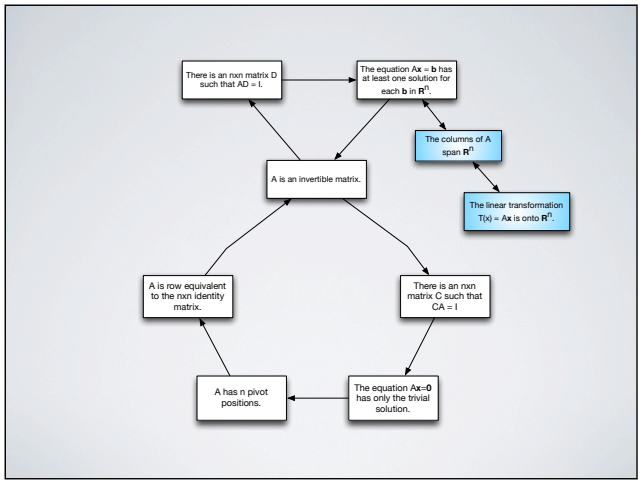
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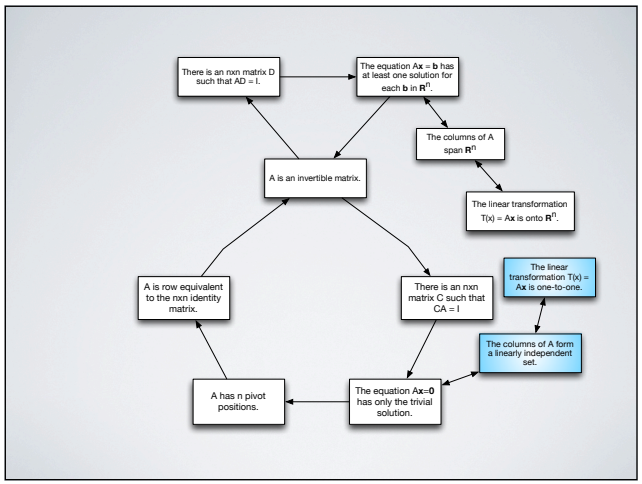
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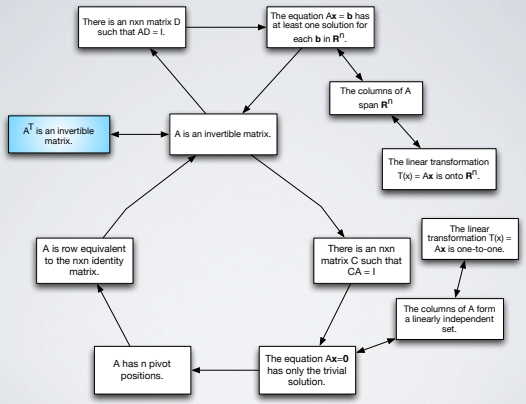

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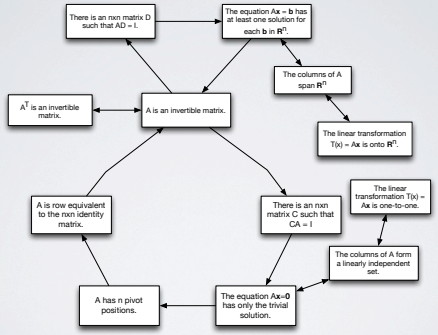
Theorem 2.8
The Invertible Matrix Theorem



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$$A = \begin{bmatrix} 8 & 9 & -1 \\ 0 & 1 & 8 \\ 5 & 4 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & -3 \\ 6 & -9 \end{bmatrix}$$

How many pivot positions in each matrix?
Are the columns linearly independent?
Is the linear transformation defined by the matrix onto? One-to-one?
Do the columns of the matrix span their "home" space?

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