MAT 233: Linear Algebra
Application Projects

# 1 Overview

## 1.1 Purpose and description

We have seen some of the wide array of applications of linear algebra ideas through in-class work and in Take Home and In Class Assessments. To complement these brief encounters with applications, the Application Project for the course will have you work in a group with a partner to explore one application in considerably more depth.

In the Application Project, you and your partner will investigate a problem in science, computing, engineering, business, or some other field that is of mutual interest and which can be addressed using techniques and concepts of linear algebra we have learned (or will learn) this semester. Then you will research the problem, learn how linear algebra is used to address it; then pose an interesting, related question and use linear algebra to actually answer your question. You will be communicating your work in a mixture of written and oral formats, culminating in a poster session held during the last day of class.

## 1.2 Learning Objectives

You will do the following in the course of this project:

1. Gain in-depth knowledge of how linear algebra is used to address an important question in an applied field.
2. Do library and internet research to learn new techniques and/or applications of linear algebra (and the field in which your problem resides) on your own.
3. Use technology to model and solve the problems and questions you encounter.
4. Work collaboratively and productively on a semi-major project in a small group, and learn how to budget time and resources accordingly.
5. Present technical information about linear algebra in a "new-to-you" setting in both written and oral form.

# 2 Components of the Project

There are three of these: The *proposal*, the *paper*, and the *poster*.

## 2.1 Proposal

For your proposal, which is due on **Friday, March 25**, your team will describe a particular problem from science, engineering, economics, computer science, or some other field that can be addressed through the use of one or more of the linear algebra techniques we have learned this semester. Your team should *not* attempt to use these techniques to actually solve the problem you describe in this

part of the project. However, you should pose one or more interesting questions relevant to your problem and make the case that these techniques could help someone answer those questions.

You are welcome to explore an interesting problem that has already been solved (a list of some suggestions appears below) or pose an original problem that has not yet been solved. You should include any data relevant to the problem you choose, or instructions for how someone could obtain such data.

The proposal is to be a two-page paper, typed and double-spaced (LaTeX is optional but recommended, especially if there is math notation involved in the proposal) with proper formatting, references, and citations. The proposal should be written as if to a fellow MAT 233 student. So you may assume that your audience is familiar with the material we have covered as a class this semester. The proposal is to be submitted electronically as an email attachment along with electronic versions of any relevant data sets.

Your proposal will be graded on the extent to which the problem you describe is a reasonable application of linear algebra covered in the course, as well as on the clarity of your writing. (This includes grammar and punctuation.) You might find the problem that catches your interest involves linear algebra that we will learn but haven't learned yet. For a list of topics that we will see in the future, please consult the calendar or the syllabus, or just ask me.

The proposal counts for 20% of the overall project grade.

I will provide you with feedback on your proposal, including the feasibility of your proposed project and ideas for enhancing it.

After you submit your proposal, you and your partner will set about actually using the linear algebra techniques from the course to address the problem posed in your proposal. This could involve a number of intermediate steps. For example, you might need to contact governmental agencies or individuals to request specific data about a situation pertaining to your problem; you might need to engage in self-teaching on the linear algebra techniques you need; you might need to brush up on programming skills or extra-mathematical subjects that are required for solving your problem. You might find you need to make simplifying assumptions on your problem or make plans to collect data. Keep in mind that this is a real-world project, and as such you have to expect the unexpected.

As you work on your problem and apply linear algebra to solve it, you will be working towards two products: the *paper* and the *poster*.

## 2.2 Paper

For your paper, which is worth 70% of your project grade and is due on **Monday, May 2**, your team will actually use the linear algebra techniques from the course to address the problem posed in your proposal. In your paper, your team should clearly explain:

- The context of the problem you have chosen;
- The mathematical model used to solve the problem (along with any assumptions made about the problem); and
- The analysis of the mathematics used to answer interesting questions about the problem.

Most papers should be at least five pages (typed, double-spaced), but the amount and type of the mathematical notation may lengthen some papers due to sheer physical space required. (For

example, if your model uses $20 \times 20$ matrices, it'll be longer than five pages.) Your paper is to be done in LaTeX.

The paper is to be aimed at the level of a fellow MAT 233 student. Thus you may assume that your audience is familiar with the material we have covered together as a class this semester. Your explanations should be clear enough that a fellow student would be able to follow them without much extra work. In particular, you should not treat your methods as a "black box"; you should provide enough explanations for your methods that a fellow student would have a good understanding of why it works the way it does.

Your paper will be graded not only on the depth of your understanding of the application you choose but also on the clarity of your explanations. Grammar and presentation will be factored into your grade.

Use MATLAB as a tool for any kind of computation, visualization, or programming needs you encounter. Any MATLAB you use should be done from M-files (including graphs) and you should submit those as M-files as part of your paper. The paper will be submitted in print, the M-files electronically.

This paper might be the first one you have ever written that addresses a technical subject rather than a subject in the humanities. Bear in mind the following characteristics of effective technical communication:

- Good technical writing is concise, to the point, and always on topic. Avoid sweeping statements ("Since the dawn of mankind...") and the temptation to ramble.
- Good technical writing is always clear about how things work and what assumptions are being made. Do not force the reader to have to do lots of work to fill in holes in your calculations.
- Good technical writing employs the rules of English grammar and mathematical notation correctly. Your textbook is a good example of an effective blend of English and math; you will be getting some additional guidelines for good mathematical writing later.

### 2.3 Poster

On **Tuesday, May 10**, in a room to be announced, we will have a poster session during which you will share your projects with your classmates and others who might want to drop by. (A general invitation will be sent around to the campus community, and it's likely that extra credit or professional development points will be given for other math students to attend. Plan on a sizeable crowd.) Your poster should be done using a science fair-style poster, preferably on a tri-fold presentation board. (These are usually available at Wal-Mart and office supply stores, and perhaps through the FC bookstore.) The poster should use text and graphics to convey very quickly the essential ideas in your project, to someone who has *not* been in the class with us this semester. Your poster, and the quality with which you present the poster, is worth 10% of your overall project grade.

During the first half of the poster session, one person from each team will stand by his or her poster while the other person wanders from poster to poster. The person standing by the poster should be prepared to explain his or her project to students who visit the poster. During the second half of the session, the teammates will switch so that everyone gets to see all the posters.

While browsing the posters, you will be given the opportunity to vote on your classmates' projects in the categories of *Most Interesting Application*, *Most Sophisticated Use of Mathematics*,

and *Most Attractive Poster*. Winners will be announced at the end of the poster session. Students with winning projects will receive bonus points to their project grades.

## 3 Academic Honesty and Citation of Sources

You must cite your sources appropriately, and a list of references must appear at the end of your paper and proposal, formatted properly. The formatting style does not matter (APA, MLA, Chicago, etc.) but you should pick one style and stay consistent with it. Be sure to format your citations (footnotes, endnotes, etc.) correctly according to the formatting style you choose. For guidelines on documenting sources, avoiding plagiarism, and other relevant writing topics, I would recommend meeting with a representative from The Write Place at least once before turning in the final version of your paper.

You should always make it clear where you obtained your ideas and not simply copy writing or big ideas from one place to another without attribution. The way to make this attribution clear is through proper citation of sources. If you are unsure about whether you are giving proper attribution to a source, please do not guess – consult the Write Place and other sources of information about writing and plagiarism. If you are tempted simply to plagiarize to save time or get a good grade, resist that temptation and talk to me directly instead. It is possible to give modest extensions to the deadlines if it means not having to write up a student team for plagiarism.

Finally, consider the quality of any source you consider using. Acceptable sources include journal articles, books (including other linear algebra textbooks), and some web sites. **Wikipedia is not an acceptable source, because no encyclopedia is an acceptable source.** Wikipedia is a good starting point for research; if you find something interesting on Wikipedia, look at the bottom of the Wikipedia page and you will usually find a list of links to reputable sources, and then use these sources instead of Wikipedia itself.

## 4 Topic Ideas

Below is a partial list of ideas for problems to investigate. This is not exhaustive. In past years, some of the best student projects were not on this list (and even of the students' own design), so please don't feel constrained by this.

- *Systems of ordinary differential equations.* Learn about systems of differential equations, how they can be used to model dynamical systems, and how linear algebra plays into their solution. Then, construct a model of a dynamical system using a system of DE's and solve it using graphical, numerical, and/or analytic methods.
- *Internet web page ranking algorithms.* Investigate a particular algorithm used to rank web pages for internet searches, such as the PageRank algorithm used by Google. Construct a limited model of a web search using an "internet" consisting of only a few web pages, using the algorithm you chose.
- *Traffic analysis.* Extend the application to traffic flow which we studied earlier in the semester to model a real traffic system – a system of roads, a computer network, etc.
- *Cryptography.* Investigate encryption systems that use linear algebra, such as the Hill cipher. Perhaps find a historical use of such a cipher, or examine how such a cipher can be broken.

- *Linear programming.* Learn about the "simplex method" and how it is used to solve certain kinds of problems involving the technique of linear programming. (This is unrelated to computer programming.) Find a business, logistics, or science problem that is approached using linear programming and extend it to a real problem using actual data.

- *Computer graphics.* Explain how different kinds of linear transformations are used in two- and three-dimensional computer graphics to manipulate "objects" in those environments.

- *Economic models.* Describe one of the Leontief input-output models and its history. Use the model to analyze a relatively simple economy, perhaps one involving actual (if simplified) data.

- *Game theory.* Find a real-world scenario that can be modeled as a two-person, zero-sum game and analyze its optimal strategies. Doing so might require showing how the game can be analyzed using linear programming (see above), then applying the simplex method to conduct the linear programming.

- *Markov chains.* Model a board game (e.g. RISK, Chutes and Ladders), sport (e.g. tennis, baseball, jai alai), or real-world situation using a Markov chain, then determine optimal strategies through an analysis of the model.

- *Discrete dynamical systems.* These are described in your book, and we have worked with them without calling them by name on several occasions this semester. Analyze a population, perhaps using actual birth, survival, and death rates, using eigenvalues.

No two groups should choose the same problem. It is possible for two problems under the same general heading to be different enough so as to merit work by two groups. But, for example, one group doing traffic analysis of one city intersection and another group doing traffic analysis of another city intersection is too similar. The problems are considered first-come, first-serve. If you submit a proposal for a problem that is, in my estimation, too similar to another problem that has already been proposed, you will be asked to try something else.

Here are some good places to find more problem ideas:

- *Scholarly journal articles from the sciences, engineering, economics, or another field.* Franklin College has extensive access to databases of scholarly journals whose articles are available as PDF's; others can be found on Google Scholar or similar research sites. Presenting some of the content of such an article (perhaps with some supporting content from one or two other sources) in a clear manner, understandable to a fellow MAT 233 student would be an appropriate way to approach this application project.

- *Publications from the Mathematical Association of America.* In addition to scholarly journal articles, the publications of the MAA — the American Mathematical Monthly, Mathematics Magazine, and the College Mathematics Journal — often have good expository articles on applications of mathematics, including linear algebra.

- *Other linear algebra textbooks.* You can find several of these in Hamilton Library and on the bookshelves of your professors.

- *Conversations with other professors.* Talk with other mathematics faculty and faculty in other disciplines for good ideas. You will be surprised (perhaps) at how many people use linear algebra. One year, Jason Jimerson (Sociology) gave a stimulating guest lecture on his use of matrices and linear algebra to analyze connections in a social network, for instance.

## 5 Credits

I am indebted to Derek Bruff of Vanderbilt University for the idea and structure of this Application Project. Some of the specific language for the specifications of the project are taken from his, with his permission.