③ ⇒ ④ : Assume $A\vec{x}=\vec{0}$ has only triv. soln.

$\boxed{\text{Prove: } A \text{ has } n \text{ pivot positions}}$

a) Show A cannot have $> n$ piv. pos.

A only has n rows, so A can't have $> n$ piv. pos's. //

b) Show A cannot have $< n$ piv. pos.

Assume # piv. pos's $< n$, then > 1 free variable. But, ⊘ b/c $A\vec{x}=\vec{0}$ has only trivial soln. Therefore, this is impossible. I.e., # piv. pos $= n$. ☑

④ ⇒ ⑤ : Assume A has n piv. pos's.

$\boxed{\text{Prove: } A \to I_n.}$

If A has n piv. pos's, its echelon form would be:

$$\begin{bmatrix} \blacksquare & * & * & * & * \\ 0 & \blacksquare & * & * & * \\ 0 & 0 & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & & \blacksquare \end{bmatrix}$$

Use piv. pos's to 0 out entries above, then rescale to get I_n.

☑