Strategy: ① ⇒ ② ⇒ ③ ⇒ ④ ⇒ ⑤ ⇒ ①

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① ⇒ ②: Assume A is invertible.

Prove: $\exists$ $n \times n$ matrix C st. $CA = I_n$.

Since A is invertible, $\exists X$ st $AX = I$

& $XA = I$.

Set $X = C$. ☐

② ⇒ ③: Assume $\exists$ $n \times n$ C st. $CA = I_n$.

Prove: $A\vec{x} = \vec{0}$ has only trivial soln.

Set up: $A\vec{x} = \vec{0}$

$CA\vec{x} = C \cdot \vec{0}$

$\vec{x} = \vec{0}$. ☐